

We are going to prove that the vector perpendicular to line  $w_1x_1 + w_2x_2 + w_0 = 0$  can be written as  $\mathbf{n} = [w_1, w_2]^T$ .

Let's assume the line  $w_1x_1 + w_2x_2 + w_0 = 0$ . Its intersection with  $x_1$  and  $x_2$  axes can be represented by the vectors  $\mathbf{v} = [-\frac{w_0}{w_1}, 0]^T$  and  $\mathbf{u} = [0, -\frac{w_0}{w_2}]^T$ , respectively.

The vector starting from  $[0, -\frac{w_0}{w_2}]^T$  and ending at  $[-\frac{w_0}{w_1}, 0]^T$  can be written as the difference between  $\mathbf{v}$  and  $\mathbf{u}$ , as  $\mathbf{a} = [-\frac{w_0}{w_1}, \frac{w_0}{w_2}]^T$ . Vector  $\mathbf{a}$  lies on the above line.

Let  $\mathbf{n} = [n_1, n_2]$  be the perpendicular vector to the line, which is also perpendicular to  $\mathbf{a}$ , therefore their inner product will be zero, i.e.,  $\mathbf{a} \cdot \mathbf{n} = 0$ .

Based on the above equation we get  $-n_1 \frac{w_0}{w_1} + n_2 \frac{w_0}{w_2} = 0 \implies \frac{n_1}{w_1} = \frac{n_2}{w_2}$ . If we assume that  $n_2 = w_2$ , then the perpendicular vector can be written as  $\mathbf{n} = [w_1, w_2]^T$ .

$\mathbf{n}$  could be any vector as long as its elements verify the association  $\frac{n_1}{w_1} = \frac{n_2}{w_2}$ .

